Catching Up Elegantly: An Algebraic Solution

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This article provides an algebraic solution to modeling a Catch-up in a non-circular manner, eliminating the need for Goal Seek/Solver functions, enabling a seamless computation and allowing for further application/expansion/derivation of such computation.

A "Catch-up" in the private equity world is commonly used as a means for a fund Manager ("Manager") to earn a fee equal to a percentage of the profit but only after the investor has received back its investment and earned a preferred return (often expressed as an internal rate of return or "IRR"). In other words, a Catch-up is a mechanism for the Manager to get compensated for their foregone incentive fees ("Carried Interest" or "Carry") due to a preferred return. In order to earn a fee equal to a percentage of the profits on an investment or series of investments, the Manager at some point (if the investment is a success) will need to catch up on the missed incentive fees at the expense of the preferred return, and this is typically accomplished by the Manager receiving a disproportionate amount of proceeds after the investor's preferred returns are met until the Manager has "caught-up."

The concept is widely understood; however, those who tried to model it in a spreadsheet may find the math rather tricky and elusive. In order to solve for the circular logic, some use "Goal Seek" or the "Solver" function in MS Excel which requires manual intervention every time an input changes, making the model cumbersome to use. With additional functionalities such as clawback or sensitivity tables, you will find yourself (or your analyst) running the "Goal Seek" hundreds of times with every input change. Such agony can be mitigated by writing a "macro," but at this point, you will have to admit that something is not quite elegant.

A small imperfection in a financial model can quickly multiply itself and make the model heavy and clunky, if not inaccurate. To avoid any unnecessary complication, formulas should be as simple as can be, and if possible, one should avoid the use of Goal Seek/Solver or any break points where manual interventions are necessary. This article provides an algebraic solution to modeling a Catch-up in a non-circular manner, eliminating the need for Goal Seek/Solver functions, enabling a seamless computation and allowing for further application/expansion/derivation of such computation.

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Concept

In order to describe the concept of Catch-up, consider the following four simple incentive fee structures:

1. 20% Carry, No Pref, No Catch-up;
2. 20% Carry, 8% Pref, No Catch-up;
3. 20% Carry, 8% Pref, 100% Catch-up; and
4. 20% Carry, 8% Pref, 50%-50% Catch-up.

Incentive Fee Structure 1: 20% Carry, No Pref, No Catch-up

In the simplest form of incentive fee, the Manager receives a percentage of profit. Let's use 20% incentive fee for illustrative purposes: if 100 is invested and 120 is returned, then four (20% of 20) goes to the Manager, and 116 is distributed to the investors. (Gross return of 20%, net return of 16%)

20% Carry, No Pref, No Catch-up

	Total (Contribution) /Distribution	Investor	Manager	Cumulative Distribution	Manager Incentive Fee % of Profit
Investment	(100.0)	(100.0)	-		
Return of Capital	100.0	100.0	-	100.0	
Remaining Distribution	20.0	16.0	4.0	120.0	20.0%
Total Profit	**20.0**	**16.0**	**4.0**	**220.0**	**20.0%**

When the returns are low, some investors may not feel comfortable with this arrangement. Let's say 100 is invested and 102 is returned after one year — the Manager is still entitled to an incentive fee of 0.4 (= 2 × 20%), diluting the gross return of 2% to a net return of 1.6%. In order to prevent paying out incentive fees on such mediocre returns, investors started to request a preferred return.

Incentive Fee Structure 2: 20% Carry, 8% Pref, No Catch-up

With a preferred return, the Manager will not see its first dollar of incentive fee until the preferred return is satisfied. If the preferred return is 8%, the first 108 goes to the investors. Afterwards, the Manager receives its incentive fee of 2.4 (= (120-108) × 20%).

20% Carry, 8% Pref, No Catch-up

	Total (Contribution) /Distribution	Investor	Manager	Cumulative Distribution	Manager Incentive Fee % of Profit
Investment	(100.0)	(100.0)	-		
Return of Capital	100.0	100.0	-	100.0	
Preferred Return	8.0	8.0	-	108.0	-
Remaining Distribution	12.0	9.6	2.4	120.0	12.0%
Total Profit	**20.0**	**17.6**	**2.4**		**12.0%**

Now, the Manager may think her fees are unfairly curtailed. While the purpose of preferred return was to avoid undeserved incentive fee on mediocre returns, now her fees are permanently impaired below 20% as illustrated above. (Mathematically speaking, as the gross return increases, the incentive fee will approach, but never reach 20%.)

Incentive Fee Structure 3: 20% Carry, 8% Pref, 100% Catch-up

Hence the "Catch-up." The Manager is now requesting that her incentive fees be restored to the original arrangement of "20% of all profits" (i.e. (120-100) × 20% = 4)in case she performs well enough. In order to achieve this, there must be a period where the Manager receives accelerated distributions until it finally reaches the equitable threshold of "20% of all profits." If 100 is invested and 120 is returned after one year, the first 108 goes to the investor, and the next two dollars go to the Manager, so that she can "Catch-up" to the original arrangement. At this point where 110 is distributed, the Manager received 20% of the profit (110-100) × 20% = 2. The remaining 10 is distributed 80%-20%, making her final incentive fee tally at 4 (i.e. (120-100) × 20%).

20% Carry, 8% Pref, 100% Catch-up

	Total (Contribution) /Distribution	Investor	Manager	Cumulative Distribution	Manager Incentive Fee % of Profit
Investment	(100.0)	(100.0)	-		
Return of Capital	100.0	100.0	-	100.0	
Preferred Return	8.0	8.0	-	108.0	-
Next 2	2.0		2.0	110.0	20.0%
Next 10	10.0	8.0	2.0	120.0	20.0%
Total Profit	**20.0**	**16.0**	**4.0**		**20.0%**

Incentive Fee Structure 4: 20% Carry, 8% Pref, 50%-50% Catch-up

The Catch-up period can be arranged so that the investor still receives distributions along with the Manager. For example, the distribution split can be "50% to the investor and 50% to the Manager" until the Manager receives 20% of all profits. If 100 is invested and 120 is returned after one year, the first 108 goes to the investor, and the next two are split half and half by the investor and the Manager. The Manager's distribution at this point is 10% of all profit (=1/10). Catch-up continues, and the next two are split half and half by the investor and the Manager. The Manager's distribution at this point is 16.7% of all profit (=2/12). Catch-up still continues, and the next two are split half and half by the investor and the Manager. The Manager's distribution at this point is 21.4% of all profit (=3/14). This means that the Catch-up was a little too excessive — 50%-50% split should have stopped between 112 and 114 and should have shifted to 80%-20% split.

The most intuitive way to model this Catch-up is to perform a "Goal Seek" in MS Excel. In the table below, one should solve for the Catch-up amount of 0.67 by fixing the Manager's Incentive Fee % at 20%; however, as the contribution/distribution amount and the

corresponding date changes, one should always re-run the "Goal Seek." Furthermore, if this calculation is part of a larger model, then the entire computation should halt and wait for the "Goal Seek" operation.

20% Carry, 8% Pref, 50%-50% Catch-up

	Total (Contribution) /Distribution	Investor	Manager	Cumulative Distribution	Manager Incentive Fee % of Profit
Investment	(100.00)	(100.00)	-		
Return of Capital	100.00	100.00	-	100.00	
Preferred Return	8.00	8.00	-	108.00	-
Next 2	2.00	1.00	1.00	110.00	10.0%
Next 2	2.00	1.00	1.00	112.00	16.7%
Next 1.34	1.34	0.67	0.67	113.34	20.0%
Next 6.66	6.66	5.33	1.33	120.00	20.0%
Total Profit	**20.0**	**16.0**	**4.0**		**20.0%**

Formula Derivation

The language that describes the Catch-up is circular in nature, as in the example below:

Cash available for distribution shall be distributed in the following order:

(i) 100% to the Investor until the Investor has received 8% annualized, compounded return on its invested capital;

(ii) 100% to the Investor until the Investor has received 100% of its invested capital;

(iii) 50% to Investor, and (B) 50% to the Manager, until the Manager has received cumulative distributions pursuant to this clause (iii) equal to 20% of all distributions made pursuant to clause (i) above and this clause (iii), and

(iv) 80% to the Investor and 20% to the Manager.

The result of the Catch-up in clause (iii) shall be includedin the calculation of clause (iii). This inherent circularity seems to suggest a simple formulaic solution is impossible; however, if we start to decompose the clause (iii), the logic can be distilled into two equations:

- Equation 1 (1a, 1b)
  Investor ("INV") distribution in tier (iii) ("INV match") is equal to Manager ("MGR") distribution in tier (iii) ("MGR-catch")

$$\sum_{n=0}^{t} INV_{match} = \sum_{n=0}^{t} MGR_{catch}$$

Equation 1a

INV match and MGR catch are equal because the split between the Investor ("INV split") and the Manager ("MGR split") is half and half in the Catch-up tier; however, this may not always be true. For example, the split can be 30% to INV and 70% to MGR. In order to capture this variability in the split, we can tweak the formula to incorporate the flexible split between LP and GP, as following:

$$\frac{\sum_{n=0}^{t} INV_{match}}{INV_{split}} = \frac{\sum_{n=0}^{t} MGR_{catch}}{MGR_{split}} \qquad \text{Equation 1b}$$

- Equation 2 (2a, 2b)
  After distribution in tier (iii) is complete, MGR catch should be 20% of the sum of all distributions in tier (ii) and tier (iii), which includes INV pref, MGR catch, and INV match. This clause can be expressed as following:

$$\sum_{n=0}^{t} MGR_{catch} = 20\% \left(\sum_{n=0}^{t} INV_{pref} + \sum_{n=0}^{t} INV_{match} + \sum_{n=0}^{t} MGR_{catch} \right) \qquad \text{Equation 2a}$$

To allow for more flexibility to the incentive fee percentage rather than fixing it as 20%, we can use "MGR%" rather than 20%, as following:

$$\sum_{n=0}^{t} MGR_{catch} = MGR_{\%} \left(\sum_{n=0}^{t} INV_{pref} + \sum_{n=0}^{t} INV_{match} + \sum_{n=0}^{t} MGR_{catch} \right) \qquad \text{Equation 2b}$$

We have two equations (i.e., Equation 1b and Equation 2b) with two variables (i.e., MGR catch and INV match). (Please note that INV split, MGR split, INV pref, MGR% are not variables, but rather a fixed number per the agreement.) In this simultaneous equation, we can delete one variable and solve for the remaining variable. First, Equation 1b can be restated in terms of INV match as in Equation 1c below:

$$\sum_{n=0}^{t} INV_{match} = \frac{INV_{split}}{MGR_{split}} \sum_{n=0}^{t} MGR_{catch} \qquad \text{Equation 1c}$$

Now, Equation 1c can be merged into Equation 2b, by replacing $\sum_{n=0}^{t} INV_{match}$ with $\frac{INV_{split}}{MGR_{split}} \sum_{n=0}^{t} MGR_{catch}$, as follows:

$$\sum_{n=0}^{t} MGR_{catch} = MGR_{\%} \left(\sum_{n=0}^{t} INV_{pref} + \sum_{n=0}^{t} INV_{match} + \sum_{n=0}^{t} MGR_{catch} \right) \quad \text{Equation 2b}$$

$$\sum_{n=0}^{t} MGR_{catch} = MGR_{\%} \left(\sum_{n=0}^{t} INV_{pref} + \frac{INV_{split}}{MGR_{split}} \sum_{n=0}^{t} MGR_{catch} + \sum_{n=0}^{t} MGR_{catch} \right)$$

$$\sum_{n=0}^{t} MGR_{catch} = MGR_{\%} \sum_{n=0}^{t} INV_{pref} + MGR_{\%} \frac{INV_{split}}{MGR_{split}} \sum_{n=0}^{t} MGR_{catch} + MGR_{\%} \sum_{n=0}^{t} MGR_{catch}$$

$$\sum_{n=0}^{t} MGR_{catch} - MGR_{\%} \frac{INV_{split}}{MGR_{split}} \sum_{n=0}^{t} MGR_{catch} - MGR_{\%} \sum_{n=0}^{t} MGR_{catch} = MGR_{\%} \sum_{n=0}^{t} INV_{pref}$$

$$\left(1 - MGR_{\%} \frac{INV_{split}}{MGR_{split}} - MGR_{\%} \right) \sum_{n=1}^{t} MGR_{catch} = MGR_{\%} \sum_{n=1}^{t} INV_{pref}$$

$$\sum_{n=1}^{t} MGR_{catch} = \frac{MGR_{\%} \sum_{n=1}^{t} INV_{pref}}{1 - MGR_{\%} \frac{INV_{split}}{MGR_{split}} - MGR_{\%}} \quad \text{Equation 3a}$$

Equation 3a solves for cumulative Catch-up amount in Tier (iii), by only using the given inputs (i.e., MGR%, MGR split, INV split) and the computation result in Tier (ii) (i.e., aggregate INV pref). Equation 3a can be further refined to capture the Catch-up amount at a given time (when n=t, rather than cumulatively), as in Equation 3b below:

$$MGR_{catch} = \frac{MGR_{\%} \sum_{n=1}^{t} INV_{pref}}{1 - MGR_{\%} \frac{INV_{split}}{MGR_{split}}} - MGR_{\%} - \sum_{n=1}^{t-1} MGR_{catch} \qquad \text{Equation 3b}$$

The mathematical notation "Σ" can be translated into spreadsheet with the use of "Sum" formula in combination with the use of $ signs to lock rows/columns.

Benefits of the Non-circular Catch-up Formula

By introducing this non-circular Catch-up formula, we eliminated the use of Goal Seek/ Solver. This enables us to run unlimited number of iterations of this section of the model without manual intervention. In practice, this means we can achieve the following:

- Change inputs and get instant outputs without interruption (with Goal Seek/ Solver, one should re-run Goal Seek/ Solver operation with each input change);
- Perform sensitivity analysis without interruption; and
- Perform clawback calculations without interruption.

In practice, Catch-up is included in the majority of private equity fund limited partnership agreements and some joint venture agreements. These agreements can include fairly complex concepts especially when dealing with multiple investments under one agreement. Being able to use a non-circular Catch-up formula enables us to create a model which can seamlessly perform a computation far more complex than (but still derived from and built upon) the example introduced in this article.